2/17/17 "Dont cry because its over, smile because it happened."-Dr. Seuss

HW: "Summation Notation" Homework section

AIM: What is Summation Notation?

to end with

SUMMATION (SIGMA) NOTATION

to start with

$$\sum_{i=a}^{n} f(i) = f(a) + f(a+1) + f(a+2) + \cdots + f(n)$$

what to do

where i is called the **index variable**, which starts at a value of a, ends at a value of n, and moves by unit increments (increase by 1 each time).

***Exercise* #1:** Evaluate each of the following sums.

End

(a) $\sum_{i=3}^{5} 2i = 2(3) + 2(4) + 2(5)$

$6 + 8 + 10 = \boxed{24}$

Start

(b) $\sum_{k=-1}^{3} k^2 = (-1)^2 + (0)^2 + (1)^2 + (2)^2 + (3)^2$

$1 + 0 + 1 + 4 + 9 = \boxed{15}$

(c) $\sum_{j=-2}^{2} 2^j = (2)^{-2} + (2)^{-1} + (2)^0 + (2)^1 + (2)^2$

$\frac{1}{4} + \frac{1}{2} + 1 + 2 + 4 = \frac{31}{4} = \boxed{7.75}$

(d) $\sum_{i=1}^{5} (-1)^i = (-1)^1 + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5$

$-1 + 1 + (-1) + 1 + (-1) = \boxed{-1}$

Put in the parenthesis!

(e) $\sum_{k=0}^{2} (2k+1) = 9$

(f) $\sum_{i=1}^{3} i(i+1) = 20$

***Exercise* #2:** Which of represents the value of $\sum_{i=1}^{4} \frac{1}{i}$?

(1) $\frac{1}{10}$ (3) $\frac{25}{12}$

(2) $\frac{9}{4}$ (4) $\frac{31}{24}$

***Exercise* #3:** Consider the sequence defined recursively by $a_n = a_{n-1} + 2a_{n-2}$ and $a_1 = 0$ and $a_2 = 1$. Find the value of $\sum_{i=4}^{7} a_i$

skip, for now.

***Exercise* #4:** Express each sum using sigma notation. Use *i* as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and sum.

(a) $9+16+25+\cdots+100$

$3^2+4^2+5^2+\ldots+10^2$

$$\boxed{\sum_{i=3}^{10} i^2}$$

(b) $-6+-3+0+3+\cdots 15$ STARTING ANSWER (not necessarily the starting #)

$3(-2)+3(-1)+3(0)+3(1)\ldots\ldots3(5)$

$$\sum_{i=-2}^{5} 3(i) \quad \text{OR} \quad \sum_{i=0}^{i=7} (-6+3i)$$

(c) $\frac{1}{25}+\frac{1}{5}+1+5+\cdots 625$

$\frac{1}{5^2}+\frac{1}{5}+1+5+\ldots\ldots$

$5^{-2}+5^{-1}+5^0+5^1+\ldots\ldots 5^4$

$$\sum_{i=-2}^{4} 5^i$$

***Exercise* #5:** Which of the following represents the sum $3+6+12+24+48$?

(1) $\sum_{i=1}^{5} 3^i$ (3) $\sum_{i=0}^{4} 6^{i-1}$

(2) $\sum_{i=0}^{4} 3(2)^i$ (4) $\sum_{i=3}^{48} i$

multiplying by 2
every time
means

$3 \cdot 2^x$

***Exercise* #6:** Some sums are more interesting than others. Determine the value of $\sum_{i=1}^{99}\left(\frac{1}{i}-\frac{1}{i+1}\right)$. Show your reasoning. This is known as a **telescoping series (or sum)**.